


EXERCISES FOR SECTION 3.7

-  **1. Numerical, Graphical, and Analytic Analysis** Find two positive numbers whose sum is 110 and whose product is a maximum.

(a) Analytically complete six rows of a table such as the one below. (The first two rows are shown.)

First Number x	Second Number	Product P
10	$110 - 10$	$10(110 - 10) = 1000$
20	$110 - 20$	$20(110 - 20) = 1800$

- (b) Use a graphing utility to generate additional rows of the table. Use the table to estimate the solution. (*Hint:* Use the *table* feature of the graphing utility.)
- (c) Write the product P as a function of x .
- (d) Use a graphing utility to graph the function in part (c) and estimate the solution from the graph.
- (e) Use calculus to find the critical number of the function in part (c). Then find the two numbers.

In Exercises 2–6, find two positive numbers that satisfy the given requirements.

2. The sum is S and the product is a maximum.
3. The product is 192 and the sum is a minimum.
4. The product is 192 and the sum of the first plus three times the second is a minimum.
5. The second number is the reciprocal of the first and the sum is a minimum.
6. The sum of the first and twice the second is 100 and the product is a maximum.

In Exercises 7 and 8, find the length and width of a rectangle that has the given perimeter and a maximum area.

7. Perimeter: 100 meters
8. Perimeter: P units

In Exercises 9 and 10, find the length and width of a rectangle that has the given area and a minimum perimeter.

9. Area: 64 square feet
10. Area: A square centimeters

In Exercises 11–14, find the point on the graph of the function that is closest to the given point.

Function	Point	Function	Point
11. $f(x) = \sqrt{x}$	$(4, 0)$	12. $f(x) = \sqrt{x - 8}$	$(2, 0)$
13. $f(x) = x^2$	$(2, \frac{1}{2})$	14. $f(x) = (x + 1)^2$	$(5, 3)$

15. **Chemical Reaction** In an autocatalytic chemical reaction, the product formed is a catalyst for the reaction. If Q_0 is the amount of the original substance and x is the amount of catalyst formed, the rate of chemical reaction is

$$\frac{dQ}{dx} = kx(Q_0 - x).$$

For what value of x will the rate of chemical reaction be greatest?

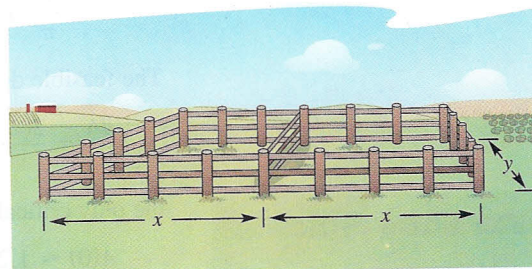
16. **Traffic Control** On a given day, the flow rate F (cars per hour) on a congested roadway is

$$F = \frac{v}{22 + 0.02v^2}$$

where v is the speed of the traffic in miles per hour. What speed will maximize the flow rate on the road?

17. **Area** A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

18. **Area** A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals (see figure). What dimensions should be used so that the enclosed area will be a maximum?



19. **Volume**

- (a) Verify that each of the rectangular solids shown in the figure has a surface area of 150 square inches.
- (b) Find the volume of each.
- (c) Determine the dimensions of a rectangular solid (with a square base) of maximum volume if its surface area is 150 square inches.

